**Cyclic Groups:**

is cyclic when consists of all powers of :

ifhas order , is cyclic group of order

and

**Cosets:**

Let be a group and let be a subgroup of

For any , is set of all products as remains fixed and ranges over

Every coset is a subset of

If cosets are equals, they must be equal sets

IF then

LaGrange’s Thm: Let be a finite group and a subgroup. The order of is a multiple of the order of .

Prime Order: If is a group with prime number elements, the is a cyclic group. Any (not equal to identity) is a generator of

is group, is subgroup (number of cosets of in

Cauchy’s thm: If is finite group, and is a prime divisor of the order of , has element of order .

The order of any element of a finite group divides the order of the group.

**Homomorphisms:**

Function s.t is a homomorphism.

is a homorphic image of

Let Then where is the identity of

is called a normal subgroup if

If is a homomorphism: kernel of is a normal subgroup of . The range of is a subgroup of

**Quotient Groups:**

Let be a group and a subgroup of :

If is normal subgroup:

Let be a group and a normal subgroup:

is a quotient group. with coset multiplication is a group. is a homographic image of

Let be a group and a subgroup. and

Commutator of is any element of the form . In albenian group, all commutators are equal to .

**FHT:**

Notions of homomorphic image and quotient group are interchangeable

Let be homomorphism with kernel :

For each , is matched with element

Let be homomorphism of onto if is kernel of :

isomorphic to the quotient group of by the kernel .

To show construct isomorphism

Define map prove well defined, homomorphism, bijection

Define map prove it is a homomorphism, surjection, and the

**Rings:**

To be a ring a set : addition must be commutative, addition and multiplication are associative, must have identity, must have additive inverse, must be distributive.

Identity: zero element 0. Additive inverse: negative Subtraction

Let be any element of Ring :

When multiplication is commutative, the ring becomes a commutative ring.

If there is an identity element for multiplication, this identity is the unity of :

An element is invertible in a ring if there is some

Divisor of zero: there is some nonzero element in the ring st

Cancellation: implies , for elements in ring and

Ring has cancellation property if it has no divisors of 0.

Field: Ring that is commutative, has unity, every nonzero element is invertible.

Integral Domain: Commutative Ring with unity having the cancellation property.

Let be a ring and a nonempty subset: If sum of any two elements of is in , is closed WRT addition. If negative of every element of is in it is closed WRT negatives. Product of any two elements of in , closed WRT multiplication.

has ideal if and :

**Ideals and Homomorphisms:**

If and are rings for to be a homomorphism

is homomorphic image of . The range of is subring of

Isomorphism from to is a homomorphisms that is injective and surjective:

The kernel of of a homomorphism is an ideal of :

**Quotient Rings**: Let be a ring and an ideal of : is a coset of

If

Consider with coset addition and multiplication is a Ring, and a homomorphic image of .

Let be a homomorphism from Ring onto Ring , then

Let be an ideal of a commutative ring, if then or . Then is a prime ideal, and is an integral domain.

**Integral Domains**:

is sum of , times. and .

The additive order of is smallest st

If is ring with unity, and has additive order , ring has characteristic

All nonzero elements in integral domain have the same additive order. The characteristic is a prime.

In any integral domain of characteristic ,

Every field is an integral domain.

Every finite integral domain is a field.

Example of Ring: set

is ring with multiplicative inverse for all elements

and are examples of fields

is an integral domain but not a field because it does not contain quotients m/n integers

is subring of

is field of quotients of

**Cyclic:**

List elements of in

**Cosets:**

Continue till you have all of

**Homomorphisms:**

**Quotient Groups:**

Continue until you have all of

Let and

Is H normal? Check subgroup

Let

Let then

**FHT:**

is surjective since if

is a homomorphism since

By FHT

Surjective since

Homomorphism since:

By FHT

**Rings:**

Prove that in any ring if then

Similar argument for

Prove that only in integral domains and are their own multiplicative terms.

(integral domain)

Thus

**Ideals and Homomorphisms:**

Show that is a subring of

is nonempty since

CWRTSub

CWRTMulti

Show that

Is ideal of

Let

Thus it Absorbs products

**Quotient Ring:**

What does QR look like when

are the three cosets of

Let be a ring and and ideal that contains Show is commutative.

Since

So

Show that

By FHT